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# Spontaneously broken supergravity: Old and new facts<sup>1</sup>

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## ABSTRACT

We report on some recent investigations of the structure of the four dimensional gauged supergravity Lagrangian which emerges from flux and Scherk–Schwarz compactifications in higher dimensions. Special attention is given to the gauge structure of M–theory compactified on a seven torus with 4–form and geometrical (spin connection) fluxes turned on. A class of vacua, with flat space–time and described by “no–scale” supergravity models, is analyzed.

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## 1 Introduction

New massive deformations of “extended” supergravity theories have recently been investigated in the context of flux compactifications from higher dimensional theories. The latter correspond to superstring or M-theory vacua with some p-form field strength turned on along the compactified directions [1]- [31]. In a more sophisticated mathematical language they correspond to fluxes when the p-form is integrated on a p-cycle in the internal manifold.

The presence of fluxes determines indeed a non-trivial scalar potential [1] in the effective low-energy supergravity, which defines in some cases vacua with vanishing cosmological constant (at tree level), in which spontaneous (partial) supersymmetry breaking may occur and (some of) the moduli of the internal manifold are fixed. In fact theories with vanishing cosmological constant are generalized no-scale models, which were studied long ago in the pure supergravity context [32, 33]. The presence of fluxes also gives rise in the low-energy supergravity to local symmetries gauged by vector fields<sup>2</sup>. Supergravity models with such gauge symmetries (gauged supergravities) have been extensively studied in the literature [34]- [35], also in connection to flux compactifications or Scherk–Schwarz dimensional reduction [25], [36]- [50]. Actually in extended supergravities ( $N \geq 2$ ) the *gauging* procedure, which consists in promoting a global symmetry group of the Lagrangian to local invariance, is the only way for introducing a non-trivial scalar potential without explicitly breaking supersymmetry. The global symmetry group of extended supergravities is the isometry group  $G$  of the scalar manifold, whose non-linear action on the scalar fields is associated with an electric/magnetic duality action on the  $n_v$  vector field strengths and their duals [51]. This duality transformation is required in four dimensions to be symplectic and thus is defined by the embedding of  $G$  inside  $\mathrm{Sp}(2n_v, \mathbb{R})$ . Gauge symmetries deriving from flux compactifications typically are related to non-semisimple Lie groups  $\mathcal{G}$  containing abelian translational isometries acting on axionic fields which originate from ten dimensional R–R forms  $C_{(p)}$  ( $p = 0, 2, 4$  for Type IIB) or the NS 2-form  $B_{(2)}$ . The embedding of  $\mathcal{G}$  inside  $G$  is defined at the level of the corresponding Lie algebras by the flux tensors themselves, which play the mathematical role of an *embedding matrix* [35].

No-scale models arising from flux compactifications or Scherk–Schwarz dimensional reduction give rise to a semi-positive definite scalar potential which has an interpretation in terms of an  $N$ -extended gauged supergravity in four dimensions. Let us recall the general form of such scalar potential  $V(\Phi)$ ,  $\Phi$  denoting collectively the scalar fields, [52–54]:

$$\delta_B^A V(\Phi) = -3S^{AC}S_{BC} + N^{IA}N_{IB}, \quad (1.1)$$

where  $S_{AB} = S_{BA}$ , and  $N^{IA}$  appear in the gravitino and spin 1/2 supersymmetry transformations

$$\delta\psi_{A\mu} = \frac{1}{2}S_{AB}\gamma_\mu\epsilon^B + \dots \quad (1.2)$$

$$\delta\lambda^I = N^{IA}\epsilon_A + \dots, \quad (1.3)$$

and give rise in the supergravity Lagrangian to the following terms:

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \dots + S_{AB}\bar{\psi}_\mu^A\sigma^{\mu\nu}\psi_\nu^B + iN^{IA}\bar{\lambda}_I\gamma^\mu\psi_{\mu A} - V(\Phi). \quad (1.4)$$

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<sup>2</sup>In four dimensional supergravities coupled to linear multiplets, fluxes may give rise to more general couplings.

Flat space demands that on the extremes  $\partial V/\partial\Phi = 0$  the potential vanishes, so

$$3 \sum_C S^{AC} S_{CA} = \sum_I N^{IA} N_{IA}, \quad \forall A, \quad (1.5)$$

The first term in the potential (1.1) is the square of the gravitino mass matrix. It is hermitian, so it can be diagonalized by a unitary transformation. Assume that it is already diagonal, then the eigenvalue in the entry  $(A_0, A_0)$  is non zero if and only if  $N^{IA_0} \neq 0$  for some  $I$ . On the other hand, if the gravitino mass matrix vanishes then  $N^{IA}$  must be zero.

For no-scale models [32, 33], there is a subset of fields  $\lambda^{I'}$  for which

$$3 \sum_C S^{AC} S_{CA} = \sum_{I'} N^{I'A} N_{I'A}, \quad \forall A \quad (1.6)$$

at any point in the scalar manifold  $\mathcal{M}_{scal}$ . This implies that the potential is given by

$$V(\Phi) = \sum_{I \neq I'} N^{IA} N_{IA}, \quad (1.7)$$

and it is manifestly positive definite. Zero vacuum energy on a point of  $\mathcal{M}_{scal}$  implies that  $N^{IA} = 0$ ,  $I \neq I'$  at that point. This happens independently of the number of unbroken supersymmetries, which is controlled by  $N^{I'A}$ .

In the sequel we shall discuss no-scale models as they originate from M-theory compactifications on a twisted seven-torus with 4-form flux, 7-form flux and geometrical flux [27]-[31], [55]-[58].

A twisted torus corresponds, in this framework, to the so called Scherk–Schwarz compactification, i.e. to the replacement of a flat torus  $T^7$  with a seven-dimensional group–manifold whose structure constants  $\tau_{IJ}{}^K$  (from now on the capital latin indices label the seven directions of the internal torus:  $I, J, M, N \dots = 1, \dots, 7$ ) determine the Lie algebra of the “graviphoton fields”  $A_\mu^I$  associated with the Kaluza–Klein mixed components of the metric  $g_{I\mu}$ . The corresponding four dimensional curvatures are therefore:

$$F^I = dA^I + \frac{1}{2} \tau_{KL}{}^I A^K \wedge A^L. \quad (1.8)$$

The internal curvature of the eleven dimensional 3-form field  $C^{(3)}$  is given by:

$$F_{IJKL}^{(0)} = -g_{IJKL} - \frac{3}{2} \tau_{IJ}{}^M C_{KL]M}, \quad (1.9)$$

while the external (space–time) components of the same field strength read:

$$F^{(4)} = d\mathcal{A}^{(3)} - g_{IJKL} A^I \wedge A^J \wedge A^K \wedge A^L - B_I \wedge F^I, \quad (1.10)$$

where  $\mathcal{A}^{(3)}$  denote the (non–propagating) four dimensional 3-form field and  $B_{\mu\nu I}$  are the seven antisymmetric tensor fields originating from the dimensional reduction of  $C^{(3)}$ . The constants  $g_{IJKL}$ ,  $\tau_{IJ}{}^K$  are bounded to satisfy the following relations:

$$\tau_{IJ}{}^J = 0; \quad \tau_{[IJ}{}^M \tau_{K]M}{}^L = 0; \quad \tau_{[IJ}{}^P g_{KLM]P} = 0. \quad (1.11)$$

These constraints ensure that, when massive antisymmetric tensors are suitably dualized to massive vector fields, so that the M-theory in  $D = 4$  admits a global (on-shell)  $E_{7(7)}$  symmetry, a 28-dimensional Lie algebra is gauged, whose structure constants are given in terms of  $g_{IJKL}$ ,  $\tau_{IJ}{}^K$  and  $\tilde{g}$  (where  $\tilde{g}$  is the flux associated with the space-time components of the 4-form:  $F_{\mu\nu\rho\sigma}^{(4)} \propto \tilde{g} \epsilon_{\mu\nu\rho\sigma}$ ). In section 2 we discuss the equations of motion and the potential of M-theory compactification on a twisted torus with internal fluxes turned on. In section 3 we discuss flat vacua and the correspondence to the Scherk–Schwarz breaking. In section 4 we discuss these results in terms of the gauging of a subalgebra of  $E_{7(7)}$ . We refer the reader to the appendix for a description of the dual gauge algebra as a subalgebra of  $E_{7(7)}$ .

## 2 The equations of motion and the potential

The bosonic equations of motion of M-theory can be obtained by varying the Lagrangian with respect to the vielbein 1-form  $V^a$  and the 3-form  $C^{(3)}$ .

The  $g_{\mu\nu}$ ,  $G_{IJ}$  and  $A^I$  field equations come from the eleven dimensional Einstein equations:

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R &= T_{\mu\nu}, \\ R_{\mu I} + \frac{1}{2} g_{\mu I} R &= T_{\mu I}, \\ R_{IJ} + \frac{1}{2} G_{IJ} R &= T_{IJ}, \end{aligned} \quad (2.1)$$

where  $g_{\mu I} = G_{IJ} A_\mu^J$  and  $G_{IJ}$  are the coordinates of  $GL(7)/SO(7)$ <sup>3</sup>. The tensor  $T$  is the energy momentum tensor of the 4-form. Incidentally we remark that in this formulation the  $R$ -symmetry of the corresponding  $N = 8$  supergravity is  $Spin(7)$ , the eleven dimensional gravitino gives rise to eight gravitinos which are in the eight-dimensional spinorial representation and to spin 1/2 which transform in the  $8 + \mathbf{48}$  of the same group.

The 3-form field equations read as follows:<sup>4</sup>

$$d \star F^{(4)} = \frac{1}{4} F^{(4)} \wedge F^{(4)}. \quad (2.2)$$

Since in this paper we are mainly concerned with the general form of the scalar potential coming from the twist and the fluxes, we will carefully analyze this equation only for those entries which receive contributions from the scalar potential [57]. Let us write the dual of the field equations originating from the Euler–Lagrange equations for  $\mathcal{A}_{\mu\nu\rho}$  and  $C_{IJK}$ . The first equation allows us to integrate out the  $\mathcal{A}_{\mu\nu\rho}$  field in a manner which we shall explain in a moment. This integration gives an extra contribution to the scalar potential coming from the Chern–Simons term. The second equation contains the derivative of the vacuum energy with respect to  $C_{IJK}$  and contributes to the equation of motion of the  $C_{IJK}$  scalar.

Let us define the following 4-D scalar quantity:

$$P = \frac{1}{\sqrt{-g}} \epsilon^{\mu_1 \dots \mu_4} F_{\mu_1 \dots \mu_4}^{(4)}, \quad (2.3)$$

<sup>3</sup>In our notations  $G_{IJ}$  is a positive definite matrix and we adopt the “mostly minus” convention for the space–time metric.

<sup>4</sup>For the eleven dimensional equations we are using the conventions and notations of reference [59]

where  $F_{\mu_1 \dots \mu_4}^{(4)}$  was defined in eq. (1.10). For the purpose of computing the scalar potential, only the  $d\mathcal{A}^{(3)}$  part of  $F^{(4)}$  will be relevant. The  $\mathcal{A}_{\mu\nu\rho}$  field equation then reads:

$$d(V_7 P) = -\frac{1}{4} F_{IJK}^{(1)} F_{PQRS}^{(0)} \epsilon^{IJKPQRS}, \quad (2.4)$$

where the field strength  $F_{IJK}^{(1)}$  is defined as follows [57]:

$$F_{IJK}^{(1)} \equiv \mathcal{D}^{(\tau)} C_{IJK} - \tau_{[I}{}^L A_{K]L} + 4 g_{IJKL} A^L, \quad (2.5)$$

the covariant derivative  $\mathcal{D}^{(\tau)}$  corresponding to the gauge connection defined by  $\tau_{IJ}{}^K$  and  $A_{IJ}$  being the 21 vector fields originating from  $C^{(3)}$ . For our purposes we shall also restrict ourselves to the  $\mathcal{D}^{(\tau)} C_{IJK}$  term in  $F_{IJK}^{(1)}$ . Equation (2.4) implies that its right hand side is a closed form. In fact the crucial ingredient is that the term  $F_{IJK}^{(1)} F_{PQRS}^{(0)} \epsilon^{IJKPQRS}$  is an exact form on the twisted torus with fluxes, and it can be written as

$$F_{IJK}^{(1)} F_{PQRS}^{(0)} \epsilon^{IJKPQRS} = -d \left( C_{IJK} (g_{PQRS} + \frac{3}{4} \tau_{[PQ}^N C_{RS]N}) \epsilon^{IJKPQRS} + \tilde{g} \right), \quad (2.6)$$

where the integration constant  $\tilde{g}$  [60] is actually related to the dual gauge algebra in the  $E_{7(7)}$  covariant formulation described in [56]. From this we get the value of  $V_7 P$  to be:

$$V_7 P = \frac{1}{4} \left( C_{IJK} (g_{LPQR} + \frac{3}{4} \tau_{[LP}^N C_{QR]N}) \epsilon^{IJKLPQR} + \tilde{g} \right). \quad (2.7)$$

Note the important identity:

$$V_7 \frac{\delta P}{\delta C_{IJK}} = -\frac{1}{4} \epsilon^{IJKLPQR} F_{LPQR}^{(0)}. \quad (2.8)$$

Let us now turn to considering the equation of motion for the  $C_{IJK}$  fields. They read:

$$\begin{aligned} \partial_\mu (V_7 \sqrt{-g} G^{I_1 J_1} G^{I_2 J_2} G^{I_3 J_3} g^{\mu\nu} \partial_\nu C_{J_1 J_2 J_3}) &= -\frac{3}{2} \frac{1}{7!} \epsilon^{\mu\nu\rho\lambda} F_{IJKP} F_{\mu\nu\rho\lambda} \epsilon^{I_1 I_2 I_3 IJKP} + \\ &\quad -\frac{1}{2} V_7 \sqrt{-g} \tau_{PQ}^{[I_1} F^{I_2 I_3]PQ}. \end{aligned} \quad (2.9)$$

By using equations (2.7) and (2.8) and the fact that:

$$\frac{\delta(F_{IJKL}^{(0)} F^{(0)IJKL})}{\delta C_{PQR}} = -3 \tau_{IJ}^{[P} F^{(0)QR]IJ}, \quad (2.10)$$

equation (2.9) can be rewritten in the form:

$$\partial_\mu (V_7 \sqrt{-g} G^{I_1 J_1} G^{I_2 J_2} G^{I_3 J_3} g^{\mu\nu} \partial_\nu C_{J_1 J_2 J_3}) = \sqrt{-g} \frac{\delta V}{\delta C_{I_1 I_2 I_3}}, \quad (2.11)$$

where the  $C_{IJK}$ -dependent part of the potential is:

$$\begin{aligned} V_C &= \frac{3}{16} \frac{1}{7!} \frac{1}{V_7} \left( C_{IJK} (g_{LPQR} + \frac{3}{4} \tau_{[LP}^N C_{QR]N}) \epsilon^{IJKLPQR} + \tilde{g} \right)^2 + \\ &\quad + \frac{1}{6} V_7 F_{IJKL}^{(0)} F_{MNPQ}^{(0)} G^{IM} G^{JN} G^{KP} G^{LQ}, \end{aligned} \quad (2.12)$$

where  $F_{IJKL}^{(0)}$  is given in eq. (1.9). One can easily compute the scalar potential in the Einstein frame by noting that

$$g_{\mu\nu} = \frac{1}{V_7^2} g_{\mu\nu}^E. \quad (2.13)$$

Therefore in this frame, the potential becomes multiplied by an overall  $(V_7)^{-2}$ .

The full scalar potential in the Einstein frame is thus obtained by adding to  $V_C$  the Scherk–Schwarz purely  $G$ –dependent part originating from the eleven–dimensional Einstein term. It is useful to write the entire potential as the following sum:

$$V = V_E + V_K + V_{C-S}, \quad (2.14)$$

where the three terms on the right hand side originate from the eleven dimensional Einstein, kinetic and Chern–Simons terms respectively, and are found to have the following expression:

$$\begin{aligned} V_E &= \frac{1}{V_7} \left( 2 G^{KL} \tau_{KJ}{}^I \tau_{LI}{}^J + G_{II'} G^{JJ'} G^{KK'} \tau_{JK}{}^I \tau_{J'K'}{}^{I'} \right), \\ V_K &= \frac{3}{16} \frac{1}{7!} \frac{1}{V_7} (g_{IJKL} + \frac{3}{2} \tau_{[IJ}^R C_{KL]R}) (g_{MNPQ} + \frac{3}{2} \tau_{[MN}^R C_{PQ]R}) G^{IM} G^{JN} G^{KP} G^{LQ}, \\ V_{C-S} &= \frac{1}{6} \frac{1}{V_7^3} \left( C_{IJK} (g_{LPQR} + \frac{3}{4} \tau_{[LP}^N C_{QR]N}) \epsilon^{IJKLPQR} + \tilde{g} \right)^2. \end{aligned} \quad (2.15)$$

Recall that in our conventions  $G_{IJ}$  is a positive definite matrix. Note that for  $\tau = g = 0$  we just get a positive cosmological constant, as noted in [60].

### 3 Flat group vacua of the potential

The scalar potential in (2.14) and (2.15) has the property that  $V_K \geq 0$ ,  $V_{C-S} \geq 0$  while  $V_E$  has no definite sign [25]. Therefore in general we may have vacua with different signs of the cosmological constant.

From inspection of the scalar potential, let us make some general comments on the possible vacua of this class of models. We start analyzing the equation  $\delta V / \delta C_{IJK} = 0$ , necessary in order to have a bosonic background with  $C_{IJK} \equiv C_{IJK}^0 = \text{constant}$ . Because of the properties (2.8) and (2.10) this is ensured by setting  $F_{IJKL}^{(0)} = 0$  or equivalently:

$$g_{IJKL} + \frac{3}{2} \tau_{[IJ}{}^P C_{KL]P} = 0. \quad (3.1)$$

As a consequence of equation (3.1), *the 4-form flux has always a vanishing contribution ( $V_K = 0$ ) to the vacuum energy*. Next we extremize the potential with respect to the volume of the torus  $V_7$ . Taking into account the dependence of the internal metric  $G_{IJ}$  on  $V_7$ , given by:

$$G_{IJ} = (V_7)^{\frac{2}{7}} \hat{G}_{IJ} ; \quad \det(\hat{G}) = 1. \quad (3.2)$$

and using the following short-hand notation:

$$\begin{aligned} b &= 2 G^{KL} \tau_{KJ}{}^I \tau_{LI}{}^J + G_{II'} G^{JJ'} G^{KK'} \tau_{JK}{}^I \tau_{J'K'}{}^{I'}, \\ a &= C_{IJK} (g_{LPQR} + \frac{3}{4} \tau_{[LP}^N C_{QR]N}) \epsilon^{IJKLPQR} + \tilde{g}, \end{aligned}$$

the two conditions  $\delta V/\delta C_{IJK} = 0 = \delta V/\delta V_7$  will reduce the expression of the potential at the minimum  $V_0$  to:

$$V_0 = -\frac{4}{3} \frac{a^2}{(V_7^0)^3} \leq 0 \text{ where } b(V_7^0)^{\frac{12}{7}} = -\frac{7}{3} a^2. \quad (3.3)$$

From the above equations we conclude that a necessary condition for a vacuum to exist is  $V_E \leq 0$  and that at the minimum  $V \leq 0$ . This excludes the existence of a de Sitter vacuum (i.e. maximally symmetric space-time geometry with positive cosmological constant).

A particular appealing class of models, which correspond to “no-scale” supergravities [32, 33], are obtained for those gaugings for which  $V = 0$ . This defines a “flat group” [25]. From equations (3.3) this implies that  $a = b = 0$ , namely that  $V_E = V_{C-S} = 0$ , and that  $V_7$  is an unfixed modulus. Condition  $V_{C-S} = 0$  in turn implies:

$$P = 0 \Leftrightarrow C_{MNR} \left( g_{IJKL} + \frac{3}{4} \tau_{[IJ}{}^P C_{KL]P} \right) \epsilon^{MNRIJKL} + \tilde{g} = 0. \quad (3.4)$$

The second equation can also be written as the following condition on  $\tilde{g}$ :

$$\tilde{g} = \frac{3}{4} C_{MNR}^0 \tau_{[IJ}{}^P C_{KL]P}^0 \epsilon^{MNRIJKL}, \quad (3.5)$$

where  $C_{IJK}^0$  is a solution of equation (3.1) and thus depends on  $g_{IJKL}$ . This equation ensures that the  $G_{IJ}$  moduli equations are the same as in the  $g = \tilde{g} = 0$  case, because the  $F$ -contribution to the energy-momentum tensor vanishes in these vacua. Condition  $V_E = 0$  on the other hand implies restrictions of the  $\tau$  matrices. These were described in the pioneering paper of ref. [25] for  $g_{IJKL} = \tilde{g} = 0$ .

Summarizing, a necessary condition for our models to admit Minkowski vacua is that the form-fluxes  $g_{IJKL}$  and  $\tilde{g}$  satisfy, besides  $V_K = 0$  also  $V_{C-S} = 0$ . If we associate the background quantities  $\tau_{IJ}{}^K$ ,  $g_{IJKL}$  and  $\tilde{g}$  with components of a larger representation of the group  $E_{7(7)}$ , it can be shown that conditions  $V_K = 0 = V_{C-S}$  amounts to stating that  $g_{IJKL}$  and  $\tilde{g}$  can be generated by acting on  $\tau_{IJ}{}^K$  by means of an  $E_{7(7)}$  transformation or, equivalently, that all the models admitting Minkowski vacua belong to the same  $E_{7(7)}$ -orbit as the model with  $g_{IJKL} = \tilde{g} = 0$  originally considered by Scherk and Schwarz, and thus share with it the same physics (mass spectrum etc...). Therefore there is an underlying hidden  $E_{7(7)}$  symmetry which is not manifest in the formulation of these models with tensor fields, but which is apparent at the level of equations of motion and Bianchi identities in the dual description of this compactification in which the antisymmetric tensor fields are replaced by scalar fields. This global symmetry however holds only if, besides the fields, the background quantities are transformed as well, and thus should not be regarded as a symmetry of the theory, but rather as a mapping between two different theories (a proper duality). This justifies *a posteriori* the aforementioned identification of the background quantities  $\tau_{IJ}{}^K$ ,  $g_{IJKL}$  and  $\tilde{g}$  with parts of an  $E_{7(7)}$  representation.

To make a concrete example, let us consider the case in which  $I = 0, i, i = 1, \dots, 6$  with  $\tau_{IJ}{}^K = \tau_{0i}{}^j$ , zero otherwise, and  $g_{IJKL} = g_{0ijk}$ , zero otherwise. In this case  $\tau_{0i}{}^j = T_j{}^i$  is chosen to be an antisymmetric matrix of rank 3 which can be set in the form:

$$T_i{}^j = \begin{pmatrix} m_1 \epsilon & 0 & 0 \\ 0 & m_2 \epsilon & 0 \\ 0 & 0 & m_3 \epsilon \end{pmatrix}; \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (3.6)$$

In this context the equation (3.1) becomes  $F_{0ijk}^{(0)} = 0$  which fixes all  $C_{ijk}$  fields but not the  $C_{0ij}$  scalars. The  $C_{0ij}$  fields give masses to the  $A_{ij}$  vector fields with the exception of the three entries

$(ij) = (1, 2), (3, 4), (5, 6)$ . Therefore three of the  $C_{0ij}$  scalar remain massless moduli. The  $G_{IJ}$ –sector gives, as discussed in reference [25], four additional massless scalars, of which two are the volume  $V_7$  and  $G_{00}$  and two other come from internal components of the metric.

If one further discusses the spectrum of the remaining fields, the six vectors  $A_{i0}$  are eaten by the six antisymmetric tensors  $B_i$  because of the magnetic mass term in the free differential algebra [57]. An additional massless scalar comes from the massless 2–form  $B_0$  and finally an additional massless vector comes from the  $A^0$  Kaluza–Klein vector. The other six  $A^i$  vectors become massive because of the twisting of the torus. We conclude that in this theory there are always eight massless scalars and four massless vectors, in agreement with [25]. The effect of turning on  $g$  and  $\tilde{g}$  is not of giving extra masses, but of shifting the v.e.v. of the  $C_{IJK}$  fields. This can be understood by an extension of the flat group where  $g$  and  $\tilde{g}$  play the role of additional structure constants. In the next section we will recover this result as well as the form of the potential, from the underlying duality symmetry of the dual formulation of the theory, in which all antisymmetric tensors  $B_I$  are dualized into scalars  $\tilde{B}^I$  and the  $E_{7(7)}$  symmetry is recovered.

## 4 The dual gauge algebra and its scalar potential

We now interpret the above result in the usual formulation of the four dimensional theory based on the flat gauging [25, 36–38]. From the results of [56] this amounts to dualizing those vector fields which participate to the anti–Higgs mechanism, in our case they are the  $A_{0i}$  1–forms, which are therefore replaced by their  $A^{0i}$  magnetic duals. The dual gauge algebra therefore contains the following 28 generators:

$$W^{ij}, W_i, Z_i, Z_0. \quad (4.1)$$

with structure constants obtained from eq. (2.13) of [56]. The first 27 generators form an abelian algebra, and the only non vanishing commutators are those involving  $Z_0$  and given by:

$$\begin{aligned} [Z_0, Z_i] &= T_j{}^j Z_j - 12 g_{0ijk} W^{jk} + \tilde{g} W_i \\ [Z_0, W^{pq}] &= 2 T_i{}^{[p} W^{q]i} - 12 g_{0ijk} \epsilon^{ijkpql} W_l \\ [Z_0, W_i] &= T_j{}^j W_j, \end{aligned} \quad (4.2)$$

where with respect to [56] the redefinition  $g \rightarrow -12 g$  was made. This algebra defines a flat subalgebra of  $E_{7(7)}$  which fits the class of models discussed by Cremmer, Scherk and Schwarz in [36] and in [37], as it was shown in [30] and in [56]. The gauged supergravity interpretation was given in [39] and the corresponding gauge algebra is the semidirect product of a  $U(1)$  by a 27–dimensional abelian algebra and is contained in the branching of  $E_{7(7)}$  with respect to  $E_{6(6)} \times O(1, 1)$ :

$$133 \rightarrow 1_0 + 78_0 + 27'_{+2} + 27_{-2}. \quad (4.3)$$

To compare with the geometrical twist we further branch  $E_{6(6)}$  with respect to  $SL(6) \times SL(2)$ :

$$78 \rightarrow (35, 1) + (1, 3) + (20, 2), \quad (4.4)$$

$$27 \rightarrow (15', 1) + (6, 2). \quad (4.5)$$

Our gauging corresponds to the following choice of the “twist matrix” (see [30] and equation (2.9) of [56]):

$$Z_0 = -\frac{2}{3} T_i{}^j t_j{}^i + g_{0ijk} t^{ijk} + \frac{1}{9} \tilde{g} t_0, \quad (4.6)$$

where we have used the notations introduced in [56]. Here  $t_i^j$  are the generators of the maximal compact subgroup of  $\mathrm{SL}(6)$ , namely  $\mathrm{SO}(6)$ , while  $t^{ijk}$  and  $t_0$  are nilpotent generators: the former belong to the  $(\mathbf{20}, \mathbf{2})$  representation in (4.4) with positive grading with respect to the  $\mathfrak{o}(1, 1)$  generator of  $\mathrm{SL}(2)$  and the latter is the nilpotent generator of  $\mathrm{SL}(2)$  with positive grading with respect to the same generator. In the same framework we now discuss the form of the scalar potential, which is expected not to depend on the dualization procedure. In the dual formulation this potential is given by [37, 39, 40]:

$$V = e^{-6\phi} \left( \frac{1}{2} (P_0 \hat{i}^j) + \frac{1}{6} (P_0 \hat{i} \hat{j} \hat{k})^2 + (P_0^0)^2 \right) = V_E + V_K + V_{C-S}, \quad (4.7)$$

where  $\phi$  is the modulus associated with the  $0^{th}$  internal dimension of compactification, the hatted indices are rigid  $\mathrm{SO}(6)$  indices, while the quantity  $P_0$  has value in the 42-dimensional non-compact part of the  $\mathfrak{e}_{6(6)}$  Lie algebra and represents the vielbein of the five-dimensional  $\sigma$ -model  $\mathrm{E}_{6(6)}/\mathrm{USp}(8)$ . It is defined as follows:

$$P_0 = (L^{-1} Z_0 L)_{\text{non-compact}}, \quad (4.8)$$

where  $L$  is the five-dimensional coset representative which, using the solvable Lie algebra parametrization of  $\mathrm{E}_{6(6)}/\mathrm{USp}(8)$ , can be directly written in terms of our scalar fields as follows:

$$L = e^{\tilde{B}^0 t_0} e^{\frac{1}{6} C_{ijk} t^{ijk}} \mathbb{E}; \quad \mathbb{E} \in \frac{\mathrm{GL}(6)}{\mathrm{SO}(6)}. \quad (4.9)$$

Direct computation shows that:

$$\begin{aligned} P_0 \hat{i} \hat{j} &= T_i^j \mathbb{E}_{\hat{i}}^{-1i} \mathbb{E}_{\hat{j}}^{-1j}, \\ P_0 \hat{i} \hat{j} \hat{k} &\propto (g_{0ijk} + \frac{3}{4} T_{[i}^n C_{jk]n}) \mathbb{E}_i^{-1i} \mathbb{E}_j^{-1j} \mathbb{E}_{\hat{k}}^{-1k}, \\ P_0^0 &\propto \epsilon^{lmnijk} C_{lmn} (g_{0ijk} + \frac{3}{8} T_{[i}^n C_{jk]n}) + \tilde{g}. \end{aligned} \quad (4.10)$$

In this language the eight massless modes come from  $\tilde{B}^0$ , three from  $C_{0ij}$ , one from  $\phi$  and three from the metric  $G_{ij}$ . The latter can be understood from the fact that under  $\mathrm{SO}(6)$  these moduli transform in the  $\mathbf{1} + \mathbf{20}'$  and the  $\mathbf{20}'$  has two vanishing weights.

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## Appendix

We can consider the  $\mathrm{E}_{7(7)}$  generators in the  $\mathrm{GL}(7, \mathbb{R})$ -basis. This corresponds to the branching:

$$\mathbf{133} \rightarrow \mathbf{48}_0 + \mathbf{1}_0 + \mathbf{35}_{+2} + \overline{\mathbf{35}}_{-2} + \mathbf{7}_{-4} + \overline{\mathbf{7}}_{+4}.$$

The coset  $E_{7(7)}/SU(8)$  can be parametrized as follows:

$$\frac{E_{7(7)}}{SU(8)} \equiv \frac{GL(7, \mathbb{R})}{SO(7)} \ltimes \text{Span}(\mathbf{35}_{+2} + \overline{\mathbf{7}}_{+4}).$$

In this basis the  $E_{7(7)}$  generators are:

$$\begin{aligned} t_M^N &\in \mathfrak{gl}(7), \\ t^{MNP}, t_{MNP}, t_P, t^P &\in \mathbf{35}_{+2} + \overline{\mathbf{35}}_{-2} + \overline{\mathbf{7}}_{+4} + \mathbf{7}_{-4}. \\ [t_M^N, t_P^Q] &= \delta_P^N t_M^Q - \delta_M^Q t_P^N, \\ [t_M^N, t^{P_1 P_2 P_3}] &= -3 \delta_M^{[P_1} t^{P_2 P_3]N} + \frac{5}{7} \delta_M^N t^{P_1 P_2 P_3}, \\ [t_M^N, t_P] &= \delta_P^N t_M + \frac{3}{7} \delta_M^N t_P, \\ [t^{N_1 N_2 N_3}, t^{P_1 P_2 P_3}] &= \epsilon^{N_1 N_2 N_3 P_1 P_2 P_3 Q} t_Q, \\ [t_M^N, t_{P_1 P_2 P_3}] &= 3 \delta_{[P_1}^N t_{P_2 P_3]M} - \frac{5}{7} \delta_M^N t_{P_1 P_2 P_3}, \\ [t_M^N, t^P] &= -\delta_M^P t^N - \frac{3}{7} \delta_M^N t^P, \\ [t_{N_1 N_2 N_3}, t_{P_1 P_2 P_3}] &= \epsilon_{N_1 N_2 N_3 P_1 P_2 P_3 Q} t^Q, \\ [t^N, t_M] &= t_M^N + \frac{1}{7} \delta_M^N t, \\ [t^M, t^{N_1 N_2 N_3}] &= -\frac{1}{6} \epsilon^{M N_1 N_2 N_3 P_1 P_2 P_3} t_{P_1 P_2 P_3}, \\ [t_M, t_{N_1 N_2 N_3}] &= -\frac{1}{6} \epsilon_{M N_1 N_2 N_3 P_1 P_2 P_3} t^{P_1 P_2 P_3} \\ [t_{M_1 M_2 M_3}, t^{N_1 N_2 N_3}] &= 18 \delta_{[M_1 M_2}^{[N_1 N_2} t_{M_3]}^{N_3]} - \frac{24}{7} \delta_{M_1 M_2 M_3}^{N_1 N_2 N_3} t, \end{aligned}$$

where  $t \equiv t_M^M$ .

The flux algebra on a twisted torus is given by a 28-dimensional Lie algebra obtained as follows:

$$\begin{aligned} [Z_M, Z_N] &= \alpha \tau_{MN}^P Z_P + \beta g_{MNPQ} W^{PQ} + \rho \tilde{g} W_{MN}, \\ [Z_M, W^{PQ}] &= \gamma \tau_{MR}^{[P} W^{Q]R} + \sigma g_{MM_1 M_2 M_3} \epsilon^{M_1 M_2 M_3 P Q R S} W_{R S}, \\ [Z_M, W_{PQ}] &= \delta \tau_{PQ}^L W_{ML}, \\ [W^{IJ}, W^{KL}] &= -\frac{\lambda}{2} \tau_{I_1 I_2}^{[K} W_{I_3 I_4} \epsilon^{L]I J I_1 \dots I_4}, \\ [W^{IJ}, W_{KL}] &= [W_{IJ}, W_{KL}] = 0, \end{aligned}$$

where  $g_{IJKL}$ ,  $\tau_{IJ}^K$  satisfy the constraints discussed in the introduction and the gauge generators read:

$$\begin{aligned} Z_M &= \theta_{M, M_1 M_2 M_3} t^{M_1 M_2 M_3} + \theta_{M, N}^P t_P^N + \theta_{M,}^N t_N = a_1 g_{M M_1 M_2 M_3} t^{M_1 M_2 M_3} + \\ &\quad a_2 \tau_{MN}^P t_P^N + a_3 \tilde{g} t_M, \\ W^{MN} &= \theta^{MN, P Q R} t^{P Q R} + \theta^{M N, P} t_P = b_1 \tau_{P Q}^{[M} t^{N]P Q} + b_2 \epsilon^{M N M_1 \dots M_4 P} g_{M_1 \dots M_4} t_P, \\ W_{MN} &= \theta_{M N,}^P t_P = c_1 \tau_{M N}^P t_P. \end{aligned}$$

The various coefficients entering the above formulas are bound to satisfy the following relations:

$$\begin{aligned} a_2 &= \alpha = \frac{\gamma}{2} ; \quad a_1 = \frac{\beta b_1}{3 a_2} ; \quad \frac{b_2}{b_1} = \frac{1}{4} \frac{a_1}{a_2} , \\ c_1 \sigma &= -2 a_2 b_2 ; \quad \frac{\lambda}{\sigma} = \frac{6\alpha}{\beta} ; \quad \delta = \alpha ; \quad a_3 = \frac{c_1}{a_2} \rho . \end{aligned}$$

Note that the gauge generators  $W^{MN}$ ,  $W_{MN}$ , as combinations of  $E_{7(7)}$  generators, are not linearly independent, but satisfy the following constraints [56]:

$$\begin{aligned} \tau_{[PQ}^N W_{R]N} &= 0 \\ b_2 \epsilon^{M_1 M_2 M_3 M_4 PQR} g_{M_1 M_2 M_3 M_4} W_{QR} &= c_1 \tau_{ST}^P W^{ST} , \end{aligned}$$

which ensures that at most 21 of them are independent, and thus that at most 28 vector fields (including the seven vectors  $A_\mu^I$ ) are involved in the minimal couplings.

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